

A galaxy–halo model for multiple cosmological tracers

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Accepted 2017 April 28. Received 2017 April 27; in original form 2016 November 1

ABSTRACT

The information extracted from large galaxy surveys with the likes of *DES*, *DESI*, *Euclid*, *LSST*, *SKA*, and *WFIRST* will be greatly enhanced if the resultant galaxy catalogues can be cross-correlated with one another. Predicting the nature of the information gain, and developing the tools to realize it, depends on establishing a consistent model of how the galaxies detected by each survey trace the same underlying matter distribution. Existing analytic methods, such as halo occupation distribution modelling, are not well suited for this task, and can suffer from ambiguities and tuning issues when applied to multiple tracers. In this paper, we take the first step towards constructing an alternative that provides a common model for the connection between galaxies and dark matter haloes across a wide range of wavelengths (and thus tracer populations). This is based on a chain of parametrized statistical distributions that model the connection between (i) halo mass and bulk physical properties of galaxies, such as star formation rate; and (ii) those same physical properties and a variety of emission processes. The result is a flexible parametric model that allows analytic halo model calculations of one-point functions to be carried out for multiple tracers, as well as providing semi realistic galaxy properties for fast mock catalogue generation.

Key words: galaxies: abundances – galaxies: luminosity function, mass function – large-scale structure of Universe – cosmology: observations.

1 INTRODUCTION

The viability of large-scale structure surveys as a cosmological probe rests on our ability to understand the connection between galaxies and the dark matter (DM) distribution that they inhabit. Galaxies are the luminous ‘tracers’ of the DM field that we actually observe, while the clustering of the invisible DM, much of which has collapsed into discrete haloes, bears the bulk of the cosmological information. Without a sufficiently accurate model of their relationship, forthcoming galaxy surveys are limited in their ability to return accurate, precision cosmological constraints (Reid et al. 2014; Clerkin et al. 2015). As a result, there has been a concerted effort to develop theoretical models of the galaxy – DM connection, and validate them against simulations and existing observations across a wide range of frequencies (e.g. Berlind et al. 2003; Zehavi et al. 2005; Zheng et al. 2009; Skibba & Sheth 2009; White et al. 2011).

A defining feature of forthcoming surveys is their coverage of large fractions of the sky. A number of the data sets will therefore cover substantially overlapping cosmological volumes, enabling cross-correlation analyses to be performed. The relationship between different galaxy samples (often seen at different wavelengths) yields valuable additional information beyond what a single tracer

can provide, both about the galaxies themselves and the large-scale matter distribution (e.g. Gaztañaga et al. 2012). Cross-correlations can be used to side-step the cosmic variance limit on certain observables, for example McDonald & Seljak (2009); to defeat otherwise difficult systematic effects (Camera et al. 2017); or to uncover the physical properties and formation processes of the galaxies themselves (Johnston et al. 2007).

While extremely promising, such analyses also require the application of a suitable galaxy–halo model – with the added complication that it must be *consistent* between tracers. This is challenging for methods designed primarily for the single-tracer case. Existing multi wavelength data are often limited in depth, or by sample variance, making it hard to empirically calibrate the models against two or more tracers simultaneously, while including the necessary correlation information between tracers.

In this paper, we construct a modular analytic model for the connection between galaxies and haloes that provides consistent predictions across multiple wavelengths and tracer populations. We achieve this by using empirical scaling relations to connect the host halo mass to bulk physical properties of the galaxies, such as stellar mass and star formation rate (SFR). These relationships are described in a statistical manner, by constructing appropriately parametrized probability distributions around the mean scaling relations (which are also parametrized). The physical properties are then related to a variety of emission processes by another set of scaling relations, each of which is relevant to some wavelength regime.

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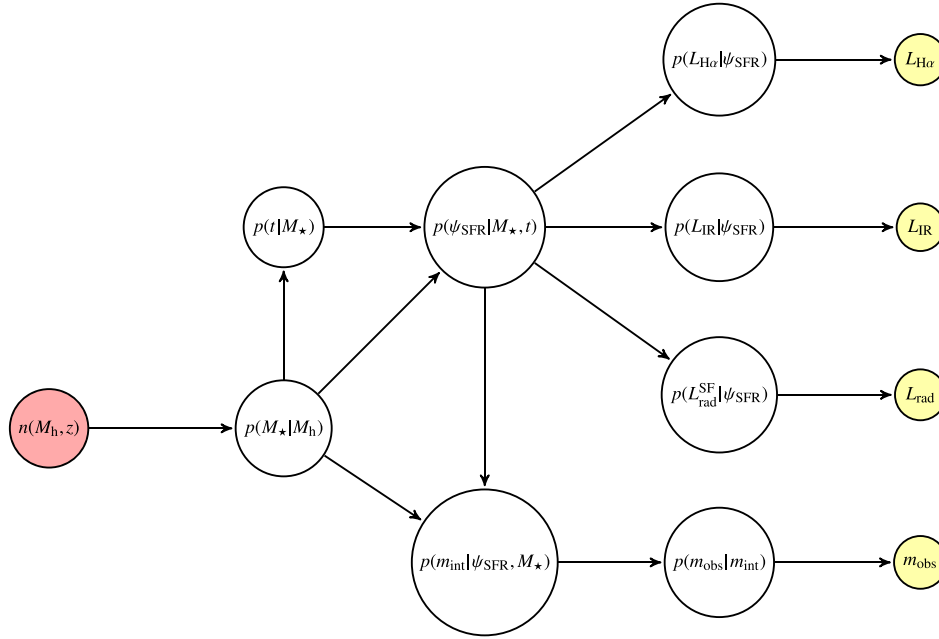


Figure 1. Graph of the probabilistic model linking input parameters (halo mass M_h and redshift z , drawn from a halo mass function) to observable luminosities/magnitudes in various bands. Only the radio and optical bands are considered in this paper; the IR and H α bands are shown for illustration.

Correlations between different types of emission (and thus different frequency bands) arise because of their shared dependence on the basic physical properties of the galaxy.

The ‘building blocks’ of the model – a chain of interconnected parametric statistical distributions (Fig. 1) – are mostly based on established relations that are found in the literature, such as the star-forming main sequence (SFMS), or stellar mass–halo mass relation. By *jointly* fitting the parameters of the model to various data sets, rather than relying on empirical calibrations of individual components from the literature, we can ensure that the components of the model all connect together in a consistent way. Each component can be replaced or upgraded as more realistic models become available, and uncertainties on model parameters can be straightforwardly propagated through the full system to determine their effects on observables, using standard Monte Carlo techniques or even analytic marginalization.

The model is differentiated from similar approaches in the literature by its analytic construction, meaning that it can be used for rapid parameter estimation and forecasting, as well as the more usual task of painting galaxy properties on to DM-only simulations to create mock catalogues. A reference implementation, written in PYTHON, is made publicly available with this paper.

As a proof of concept, in this paper we will construct a model for optical and radio continuum emission from ‘normal’ [non-active galactic nucleus (AGN)] galaxies at $z \simeq 0$, and calibrate it off multi wavelength luminosity function (LF) data (plus information from a semi-analytic simulation for one model component). Simple extensions of the model can be made to generalize it to higher redshifts and several other tracer populations. Other novel features include a new set of analytic fitting functions that connect optical magnitudes to SFR and stellar mass, and a simple dust attenuation model.

This represents only an initial step in the development of a fully viable multi tracer galaxy–halo model. Our ultimate goal is to construct a halo model with sufficient complexity to describe and predict the joint one-point (LF) and two-point (clustering) statistics of

multiple galaxy populations observed across the full range of wavebands covered by forthcoming large cosmological surveys like *Euclid*, *LSST*, *SKA*, and *WFIRST*. As discussed below, this will require significantly more complexity than the single-tracer halo occupation distribution (HOD) models currently in use, but hopefully significantly less complexity than semi-analytic modelling (SAMs) or hydrodynamical simulations. This is a reasonable goal, as the aim is only to accurately describe the general properties of *populations* of galaxies, rather than detailed properties of individual galaxies.

The paper is organized as follows. In Section 2, we briefly review various approaches that have been used to model galaxy populations. We then define the components of our model in Section 3, and calibrate it against simulations and observations in Section 4. We also analyse how well the model fits the input data, and discuss some of the other observables it can predict. We then conclude in Section 5 with a discussion of the model’s limitations, and ways that it could be improved and extended.

We denote the natural logarithm by \log and the base-10 logarithm by \log_{10} throughout, and assume the Planck Collaboration XIII (2016) best-fitting Λ cold dark matter cosmology with $h = 0.67$ and $\Omega_M = 0.32$.

2 GALAXY POPULATION MODELLING

In this section, we briefly review some of the methods for modelling galaxy populations. These generally fall into one of three main categories:

- (i) Hydrodynamical simulations;
- (ii) Semi-analytic models;
- (iii) Halo model approaches.

Hydrodynamical simulations (e.g. Hopkins et al. 2014; Vogelsberger et al. 2014) are typically used to make samples of galaxies with highly realistic properties. By explicitly modelling the physical processes relevant to galaxy formation and evolution, very detailed simulations can be constructed, albeit at very high

computational expense. This typically limits the simulations to small spatial volumes, so that sufficiently high spatial resolutions can be achieved to model the necessary processes. ‘Sub-grid’ models can be used to incorporate processes that happen on unresolvable length-scales, although these typically introduce a number of free parameters that must be tuned to match observations. While useful for building a detailed understanding of galaxies, hydrodynamical simulations are generally too expensive to use for applications such as creating large numbers of mock galaxy catalogues for the analysis of large-scale structure surveys.

SAMs are commonly used to populate DM-only N -body simulations with galaxies. They also work by explicitly modelling the various physical processes responsible for determining observable galaxy properties, subject to some simplifying assumptions designed to reduce computational overhead (Somerville & Primack 1999; Kauffmann & Haehnelt 2000; Springel et al. 2005; Benson 2012; Baugh 2013). The simplified physical models in SAMs often have tunable parameters that can be calibrated against observations. While intended to be cheaper than hydrodynamical simulations but much more realistic than halo model approaches, SAMs are still computationally intensive due to the need to repeatedly solve systems of differential equations to determine the properties of each galaxy. This makes them unwieldy for performing Monte Carlo parameter studies, for example, which require many different realizations of the galaxy population to be constructed for different sets of parameter values (although such studies have been attempted; see e.g. Henriques et al. 2009). Minimal SAMs do exist, which can be used to more rapidly model some subset of galaxy properties (e.g. Cohn 2016).

Halo model approaches are a much lighter alternative than either of the previous two methods. These work by assigning galaxy properties to DM haloes, using a set of scaling relations that depend on properties such as the halo mass. Halo models have the advantage of often being simple enough to evaluate analytically, making them suitable for cheaply populating mock catalogues (e.g. Manera et al. 2013; Koda et al. 2016), or performing statistical analyses (e.g. Reid et al. 2014).

Halo model methods come in a number of different flavours. HOD (Peacock & Smith 2000; Berlind & Weinberg 2002; Berlind et al. 2003) modelling predicts the mean number of galaxies of a given type that reside in a halo of given mass, $\langle N \rangle(M_h)$. HODs are typically calibrated empirically, with the resulting fits being extrapolated to model the distribution of the same type of galaxies in future surveys. While HODs can be simultaneously defined for multiple galaxy populations (e.g. Krumpke, Miyaji & Coil 2010; Miyaji et al. 2011; Krause et al. 2013), the correlation between each pair of populations must be modelled (and calibrated) separately, e.g. using the observed cross-correlation function. Ambiguities may also arise when trying to ascertain which galaxies appear in more than one survey – HODs calibrated at different wavelengths can predict different numbers of galaxies to exist in a given halo, so some galaxies will be ‘missing’ in one band, but not in another.

An alternative approach is to construct conditional LFs (Yang, Mo & van den Bosch 2003; Vale & Ostriker 2004; Cooray 2006), which describe the expected luminosity distribution of galaxies hosted in haloes of a given mass. These methods are again calibrated empirically against a given population of galaxies, but can be extended to other wavelengths by assuming an appropriate spectral energy distribution (SED). Correlations between different types of tracer must again be added by hand though.

A unified framework for implementing halo model methods such as these is presented in Hearin et al. (2016).

Several recent works have explicitly sought to model galaxy populations across multiple wavelengths. Some are empirical, in the sense that they create realizations of mock galaxy samples that are consistent with a set of observed LFs by construction (Jouvel et al. 2009; Schreiber et al. 2016). The data for these are generally obtained from deep but narrow surveys (e.g. COSMOS), so the redshift dependence and faint end of the LF are well measured, but large-scale clustering information tends to be minimal. A related approach is to use machine learning techniques to create a predictive model of a multi wavelength galaxy sample, again by training it off an existing galaxy catalogue (Xu et al. 2013).

Alternatively, one can take scaling relations between galaxy properties (e.g. SFR and stellar mass) and combine them to reproduce observables. This is the basic approach taken in this paper, and was also used by van den Bosch, Yang & Mo (2003); Skibba & Sheth (2009); Li et al. (2016); Schreiber et al. (2016), and others. This has the advantage of allowing several different observables to be constructed from a common set of components, and is justified by the empirical discovery of a number of appropriate scaling relations. A possible problem is inconsistency between the various components, which may be calibrated off different data sets with different assumptions. We will avoid this problem here by performing global model fits, rather than calibrating components individually.

3 MODEL DEFINITION

In this section, we specify each component of the model in detail, according to the basic structure illustrated in Fig. 1. The rationale behind this structure is that any joint probability distribution (e.g. of galaxy properties) can be decomposed into a chain of conditional distributions of the form $p(x_1, x_2, \dots, x_n) = p(x_1) \cdot p(x_2|x_1) \cdot \dots \cdot p(x_n|x_1, x_2, \dots, x_{n-1})$. Subsets of the conditional distributions can be grouped together by rewriting them as joint distributions over a subset of the variables, which in many cases can be approximated using simple analytic distributions, or even products of univariate distributions if the variables are essentially independent. This can vastly simplify the problem of modelling the full multivariate distribution if an appropriate simplifying restructuring can be found. The use of conditional distributions as tractable ‘building blocks’ of complicated joint distributions is widespread in other contexts, e.g. in Gibbs sampling methods (Casella & George 1992).

Using this approach, we apply known empirical relations between galaxy properties, plus some simple physical reasoning, to arrive at the simplified probabilistic model shown in Fig. 1. This is by no means unique, and in its present form is unable to account for some known correlations between galaxy properties (e.g. galaxy colours). These limitations would likely be exacerbated when considered clustering information, and so the model structure will need to be revisited in future. Nevertheless, as we will show below, the current structure is sophisticated enough to be able to model the joint LFs of multiple galaxy populations at radio and optical wavelengths – a non-trivial application that would normally be the preserve of SAMs.

3.1 Halo mass function

The initial input to the model is a distribution of DM haloes in mass, position, and redshift. This is represented by the halo mass function,

$$n(M_h, z) \equiv \frac{dN(z)}{d \log M_h dV dz}, \quad (1)$$

where M_h is the halo mass and V is the comoving volume. This distribution ultimately sets the overall abundance and clustering properties of galaxies, and the dependence on the underlying cosmological model. As a simplifying assumption, we will assume that DM haloes are characterized solely by their mass and redshift, ignoring other intrinsic properties such as angular momentum and concentration. Furthermore, we will make no explicit distinction between parent and sub-haloes, although in reality some galaxy properties do depend on this. Finally, we will assume that each halo (or sub-halo) hosts precisely one galaxy – there are no empty haloes, and no multiply occupied ones (in contrast to HOD, where satellites are explicitly modelled).

In most of what follows we will use an analytic form for the halo mass function, but it is important to note that the model can be applied to *any* representation of a set of DM haloes. For example, the chain of statistical distributions shown in Fig. 1 can just as well be applied to a DM halo catalogue taken from an N -body simulation, and used to populate mock catalogues of galaxies using a Monte Carlo method.

For the analytic calculations in the remainder of the paper, we will adopt the Tinker et al. (2008) mass function,

$$n(M_h, z) = -f(\sigma) \frac{\rho_m}{M_h} \frac{d \log \sigma(M_h)}{d \log M_h} \quad (2)$$

$$f(\sigma) = A \left[1 + \left(\frac{\sigma(M_h)}{b} \right)^{-a} \right] \exp(-c/\sigma^2), \quad (3)$$

where ρ_m is the background matter density and $\sigma(M)$ is the rms density fluctuation within a sphere of radius $R = (3M/4\pi\rho_m)^{1/3}$, from linear theory. The dependence on cosmological parameters is implicit in $\sigma(M)$, an integral over the matter power spectrum, $P(k)$, which we take from CAMB (Lewis, Challinor & Lasenby 2000). The best-fitting parameters, calibrated against simulations, are $A = 0.186$, $a = 1.47$, $b = 2.57$, and $c = 1.19$, all at $z = 0$.

For calculations of the clustering, one must also specify a halo bias function, $b(M_h)$. The (Eulerian) halo bias is

$$b(M_h) = 1 + (a\nu_1 - 1)/\delta_1 + \frac{2p/\delta_1}{1 + (a\nu_1)^p}, \quad (4)$$

where $\nu_1 = [\delta_1/\sigma(M_h)]^2$ and $\delta_1 \approx \delta_c$.

3.2 Stellar mass–halo mass relation

As a first step, it is necessary to link DM haloes to their baryonic content. We model this through the relationship between halo mass and stellar mass, as described by the conditional mass function. This is related to the conditional probability to find a stellar mass of M_* in a halo of mass M_h : $p(M_*|M_h)$.

Halo mass and stellar mass are expected to be strongly related as a simple consequence of hierarchical structure formation; as more massive haloes collapse, they trap a correspondingly larger mass of baryonic matter, some fraction of which forms stars as the gas settles and cools in the centre of the halo potential (White & Rees 1978). A relatively tight relationship between the two is indeed seen observationally (Yang et al. 2007; Moster et al. 2010; Behroozi, Conroy & Wechsler 2010), and is well fit by a broken power law,

$$\overline{M_*}(M_h) = \frac{2M_h A_*}{\left(\frac{M_h}{M_*^*}\right)^{-\beta_*} + \left(\frac{M_h}{M_*^*}\right)^{\gamma_*}}, \quad (5)$$

where A_* is the overall normalization, M_*^* is a mass scale, and β_* and γ_* are the low- and high-mass end slopes, respectively. The conditional pdf is modelled as log normal,

$$p(M_*|M_h) = \frac{1}{\sqrt{2\pi}M_*\sigma_{M_*}} \exp\left(-\frac{\log^2(M_*/\overline{M_*})}{2\sigma_{M_*}^2}\right), \quad (6)$$

where σ_{M_*} is the scatter, modelled by Moster et al. (2010) as

$$\frac{\sigma_{M_*}}{\log 10} = \sigma_\infty^* + \sigma_1^* \left(1 - \frac{2}{\pi} \arctan[\xi_* \log_{10}(M_h/M_*^*)]\right). \quad (7)$$

While Moster et al. (2010) provide separate fits for both central and satellite galaxies, we use only the central relation as, for simplicity, our model ignores this distinction.

3.3 Star formation rate–stellar mass relation

The SFR, ψ_{SFR} , is a key quantity that predicts many of a galaxy's other observable properties. It is thought to depend on a number of factors, including the availability of cold molecular gas (the raw material of star formation); the recent merger history of the galaxy (mergers disturb cold gas, inducing its collapse to form stars); and quenching processes such as feedback from accretion on to the central supermassive black hole or supernovae. See Somerville, Popping & Trager (2015) and references therein for a more thorough overview.

As we are only interested in the aggregate properties of a large galaxy sample, we adopt stellar mass as a sufficiently robust predictor of the SFR. A connection between the two is well established – an SFMS is found in the $\psi_{\text{SFR}} - M_*$ plane, along which the bulk of actively star-forming galaxies lie (Daddi et al. 2007; Elbaz et al. 2007; Noeske et al. 2007; Salim et al. 2007; Peng et al. 2010; Karim et al. 2011; Wang et al. 2013). A second population of *quiescent* or *passive* galaxies lies below the SFMS, which typically contain older, redder stars, and less (but not necessarily negligible) star formation. This population may be arranged in a similar, but more scattered, sequence to the SFMS, or else in a much looser, relatively uniform distribution in SFR below the main sequence; see Lagos et al. (2011) for a comparison of SAM predictions. Other populations may be identified in the $\psi_{\text{SFR}} - M_*$ plane, such as starburst galaxies (which lie above the SFMS), but we will work only with the simple active versus passive categorization here.

Motivated by the above, we will model the distributions of passive and SFMS galaxies separately. The first step is to assign a probability that a galaxy will belong to one population or the other. Several studies have attempted to estimate the fraction of passive galaxies from recent surveys (e.g. Fontana et al. 2009; Peng et al. 2010; Moustakas et al. 2013); we adopt a modified version of the relation from Behroozi, Wechsler & Conroy (2013), which they attributed to Brammer et al. (2011):

$$f_{\text{passive}}(M_*) = c + \frac{1 - c}{1 + \left(\frac{M_*}{10^{a_f} M_\odot}\right)^{\beta_f}}, \quad (8)$$

where we reparametrize $c \equiv \frac{1}{2}(1 + \tanh \zeta_f)$ to ensure that $f_{\text{passive}} \in [0, 1]$. Factors other than stellar mass can also be important for determining whether a galaxy is passive or not (e.g. environment; see Peng et al. 2010). We assume that these factors have been marginalized over here, leaving M_* as the only relevant variable. The resulting pdf, $p(t|M_*)$, where t denotes the galaxy type, is a uniform distribution partitioned at f_{passive} .

For the SFMS, we adopt a mean relation with a similar form to the one found in Wang et al. (2013),

$$\bar{\psi}_{\text{SFR}}(M_*) = 10^{\alpha_{\text{SFMS}}} \left(\frac{M_*}{10^{10} M_{\odot}} \right)^{\beta_{\text{SFMS}}} M_{\odot} \text{yr}^{-1}. \quad (9)$$

The scatter in the SFMS is incorporated by modelling it as a log normal distribution,

$$p_{\text{SFMS}}(\psi_{\text{SFR}}|M_*) = \frac{1}{\sqrt{2\pi} \psi_{\text{SFR}} \sigma_{\text{SFMS}}} \exp \left(-\frac{\log^2(\psi_{\text{SFR}}/\bar{\psi}_{\text{SFR}})}{2\sigma_{\text{SFMS}}^2} \right). \quad (10)$$

There is more freedom in deciding how to model the passive population. The results of Wang et al. (2013) suggest a relatively uniform distribution of galaxies below the SFMS, while Moustakas et al. (2013) show a similar log normal distribution to the SFMS, but shifted to higher M_* and lower ψ_{SFR} . SAMs show an even wider range of behaviours, as discussed in Lagos et al. (2011). For simplicity, we will use a log normal distribution of the same form as equation (10), but with the mean from equation (9) shifted by some factor, and a different scatter,

$$\begin{aligned} \bar{\psi}_{\text{SFR}} &\rightarrow a_{\text{pass}} \bar{\psi}_{\text{SFR}} \\ \sigma_{\text{SFMS}} &\rightarrow \sigma_{\text{pass}}. \end{aligned} \quad (11)$$

This more closely follows the observations of Moustakas et al. (2013), with the apparent shift in M_* caused by the transition $f_{\text{passive}} \rightarrow 1$ as M_* increases.

3.4 Star formation–luminosity relations

Star formation activity releases energy through several processes that cause (typically bright) emission across many bands (Kennicutt 1998). Young, high-mass stars emit ultraviolet (UV) radiation, which excites gas in the surrounding inter stellar medium (ISM). As well as the UV emission itself, this generates line radiation, for example through the de-excitation of hydrogen through $H\alpha$ and $H\beta$ emission. Young stars are short-lived, exploding as supernovae on time-scales of order 10 Myr. Supernovae inject high-energy electrons into the ISM, which interact with the galactic magnetic field to emit synchrotron radiation at radio frequencies (Condon 1992). The process of star formation itself takes place in dense clouds of gas and dust, and the latter, warmed by the collapse process, emits in the infrared.

Star formation-related emission dominates the luminosity of many galaxies. This has led to the development of many ‘SFR indicator’ relations (Kennicutt 1998; Bell 2003; Moustakas, Kennicutt & Tremonti 2006) that can be written in the form $L \approx A \psi_{\text{SFR}}^{\beta}$, with $\beta \approx 1$. In this paper, we will consider only one SFR-related type of emission as an example. This is continuum radio emission, from synchrotron radiation associated with supernova remnants. Continuum surveys with the SKA and its precursors are expected to detect tens of millions of galaxies from $z \sim 0$ –5, which can be used for weak lensing and 2D clustering studies (Jarvis et al. 2015). Assuming a power-law spectrum for the emission, $S \propto (\nu/\nu_{\text{ref}})^{\alpha}$, with $\alpha \simeq -0.7$, one can use the SFR indicator relation at 1.4 GHz from Bell (2003) to obtain

$$L_{1.4\text{GHz}} = 1.812 \times 10^{28} \left(\frac{\psi_{\text{SFR}}}{M_{\odot} \text{yr}^{-1}} \right) \text{erg s}^{-1} \text{Hz}^{-1}. \quad (12)$$

We will omit scatter in this relation, treating the scaling as being deterministic for simplicity. Unlike some other SFR indicators

(e.g. $H\alpha$; Kennicutt 1998; Pozzetti et al. 2016), the radio emission does not suffer from extinction, but can be contaminated by (e.g.) free–free and emission from AGN jets. We assume here that free–free is sub-dominant, and that AGN-dominated and passive galaxies can be selected out, leaving only normal star-forming galaxies.

3.5 Optical magnitude relations

The connection between bulk galaxy properties and emission in optical bands is more convoluted. Optical emission is primarily sourced by the aggregate of all of the stars in the galaxy, which form over an extended period of time, and evolve at different rates depending on their initial mass. The observed optical luminosity is therefore a probe of the star formation history of the galaxy, not just its present state.

SAMs typically reconstruct the star formation history explicitly, evolving an assumed initial mass function forward in time with a stellar population synthesis (SPS) model, and taking into account later bursts of star formation caused by mergers and other events. In the absence of the actual merger history from a simulation, Monte Carlo realizations of plausible merger histories can also be substituted (e.g. Parkinson, Cole & Helly 2008).

A similar approach would be possible here, at the cost of adding significant computational complexity and relying on a ‘black box’ SPS code. Instead, we attempt a simpler approximate treatment to preserve the analytic, parametric nature of the model. We begin by hypothesizing that the stellar population can be characterized by two main components: newly formed stars, dominated by the high-mass end of the initial mass function (IMF); and an older population that formed long before, dominated by lower mass stars on the main sequence. The former will be bluer, the latter redder.

Next, we propose an ansatz for the mean absolute magnitude of a galaxy in a given optical band (labelled by ν):

$$\begin{aligned} \bar{m}_{\nu}(\psi_{\text{SFR}}, M_*) = & -c_0^{(\nu)} + A_{\star}^{\text{opt}} \left(c_{\star}^{\text{opt}} + \left(\frac{M_*}{10^9 M_{\odot}} \right)^{\beta_{\star}^{\text{opt}}} \right) \\ & + A_{\times}^{(\nu)} \left(\frac{M_*}{10^9 M_{\odot}} \right)^{\beta_{\times}^{\text{opt}}} \left(\psi_{\text{SFR}} \right)^{\gamma_{\times}^{\text{opt}}}. \end{aligned} \quad (13)$$

A rough physical interpretation of each term is as follows. The first is an offset that determines a characteristic galaxy magnitude in a given band. The second characterizes the total emission from the older stellar population, using stellar mass as a proxy. The final term represents the contribution from the younger population, with SFR as a proxy. Since there is a relationship between SFR and stellar mass, this term must also include an M_* -dependent factor to model the ‘mixing’ between the two. The amplitude of the final term also depends on the band; star formation contributes less to the total luminosity in redder optical bands.

To calibrate the relation, we use the public semi-analytic catalogues of Guo et al. (2011), which provide *ugriz* absolute magnitudes with and without a dust extinction correction. The residuals of the best fitting to the unattenuated *u*-band magnitudes at $z = 0$ are shown in Fig. 2, as a function of M_* and ψ_{SFR} . The distribution of residuals is reasonably simple, with a mean close to zero and only a low level of structure present. The standard deviation of the residuals across the whole plane is $\sigma(\Delta m_{\nu}) \sim 0.3$ mag in all five bands, which is quite small considering the simplicity of this approach; it has only five global parameters and two free parameters per band. Ignoring the dependence of the residuals on M_* and ψ_{SFR} , we find

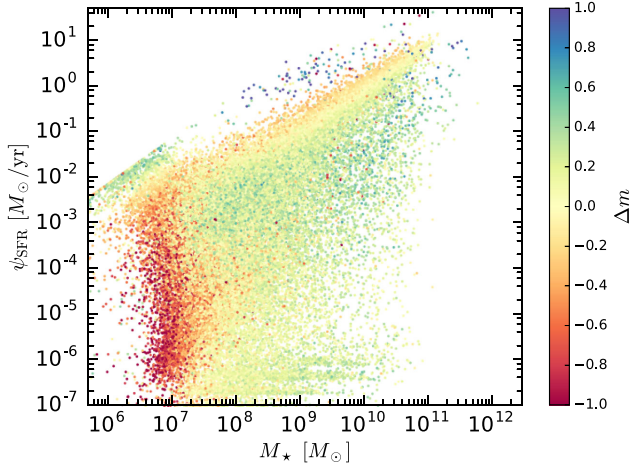


Figure 2. Difference between u -band magnitudes at $z = 0$ (without dust attenuation) in the Guo et al. (2011) simulation, and those predicted by our best-fitting relation, equation (13), as a function of SFR and stellar mass. Resolution effects in the simulation are apparent at high and low mass, and low SFR.

that they are well modelled by a shifted log normal distribution,

$$p(m_v | \psi_{\text{SFR}}, M_*, z) = \frac{1}{\sqrt{2\pi} \sigma_m^{(v)}} \exp \left(-\frac{1}{2} \left(\frac{\log x - \mu^{(v)}}{\sigma_m^{(v)}} \right)^2 \right), \quad (14)$$

where $x \equiv \bar{m}_v - m_v + b_v$, $x > 0$. Here, $\log b_v \equiv \mu^{(v)} + (\sigma_m^{(v)})^2/2$, where b_v is the mean of the shifted log normal distribution and $\mu^{(v)}$ and $\sigma_m^{(v)}$ are free parameters in each band. These parameters are calibrated by performing a least-squares fit of a log normal pdf to the histogram of residuals, independent of ψ_{SFR} and M_* .

As shown in Fig. 2, the residuals do have some dependence on ψ_{SFR} and M_* . The residuals can also be correlated between bands. While we have neglected structure like this in our model as a simplifying assumption, such correlations are partially responsible for the observational patterns seen in ‘colour–magnitude’ diagrams, which are often used to select different populations of galaxies. In its current state, the model will therefore be unable to reproduce features such as the red sequence or blue cloud (e.g. Strateva et al. 2001), for example.

3.6 Optical dust extinction

Some fraction of the optical emission from galaxies is absorbed by the dust that they contain. The amount of absorption depends on a number of factors, including wavelength (bluer colours are more strongly absorbed); morphology and inclination angle (light from the bulge and disc components is affected differently); and the dust content and its distribution inside the galaxy. Intrinsic dust absorption is difficult to simulate or characterize with observations, which is problematic – accounting for dust is a necessary step in connecting optical emission to the physical properties of galaxies, as well as being important for modelling infrared luminosities.

There have been numerous attempts to construct attenuation models from observations and simple considerations such as galaxy morphological properties (e.g. Tuffs et al. 2004; Driver et al. 2007). Several dust models that have been used in semi-analytic simulations are compared in Fontanot et al. (2009). Many such models use an empirically determined spectral dependence from Calzetti et al. (2000), which has been found to offer a good description

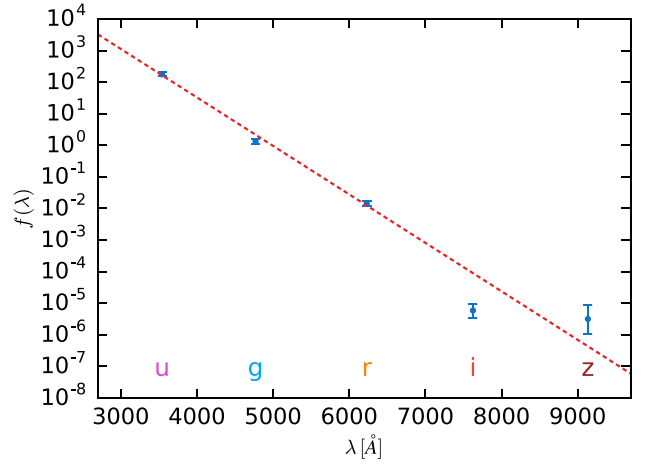


Figure 3. Amplitude of the optical dust attenuation as a function of wavelength, $f(\lambda)$. The blue data points show the median and 68 per cent credible intervals for $f(\lambda_j)$ in each band j , after conditioning on the best-fitting values of all other parameters. The red dashed line shows the relation from equation (16) for the best-fitting values of τ_0 , κ , and λ_0 .

of the corrections required to get SFR determinations at different wavelengths to agree (e.g. Pannella et al. 2009).

In keeping with our analytic approach, we will adopt a simpler parametric model. An ansatz for the optical depth due to internal dust extinction is

$$\tau(\lambda) = f(\lambda) \left(\frac{M_*}{10^{10} M_\odot} \right)^{\beta_\tau} (1 + a_{\text{disc}} \sin \theta), \quad (15)$$

where $f(\lambda)$ is the attenuation amplitude as a function of (rest-frame) wavelength, a_{disc} is the effective attenuation due to the disc, and θ is the inclination angle of the disc with respect to the line of sight. The frequency dependence (see Fig. 3) can be modelled approximately as

$$f(\lambda) = \tau_0 \exp[-\kappa(\lambda - \lambda_0)], \quad (16)$$

where τ_0 is an overall normalization parameter, κ determines the scaling with wavelength, and λ_0 is a reference wavelength. The change in magnitude due to the dust attenuation is

$$\Delta m \equiv m_{\text{obs}} - m_{\text{int}} = 5 \log_{10} \left(\frac{S_{\text{int}} e^{-\tau}}{S_{\text{int}}} \right) = 1.086 \tau, \quad (17)$$

where S_{int} is the intrinsic flux before attenuation.

The two factors in parentheses in equation (15) are motivated as follows. First, the amount of dust in the galaxy is assumed to scale with the stellar mass to some power. Dust builds up in the ISM as successive populations of stars evolve and die. While star formation is associated with thermal dust emission, this only represents one component of the total dust content of a galaxy; stellar mass should be more indicative of the integrated star formation history that has actually generated the dust. There is observational support for a relatively steep power-law scaling of the dust attenuation with stellar mass (e.g. Pannella et al. 2009; Buat et al. 2012; Heinis et al. 2014), but we will not enter into a discussion here.

Second, extinction will be greater when the disc is seen edge-on, i.e. when $\theta = \pi/2$. The relative optical depth of the disc (compared to the bulge) is represented by a scaling factor, a_{disc} . For random galaxy orientations, $\sin \theta$ follows a uniform distribution. If we assume that inclination angle is the only random variable in the optical

depth expression, it follows that

$$p(m_v^{\text{obs}} | m_v^{\text{int}}) = \text{Uniform}[m_v^{\text{int}} + \Delta m(\theta = 0), m_v^{\text{int}} + \Delta m(\theta = \pi/2)].$$

In the absence of explicit modelling of other stochastic contributions to the optical depth, it is likely that the inclination angle term will absorb other sources of scatter in the observations. As such, the a_{disc} parameter should not be interpreted directly as a physical quantity.

Since one only ever observes the dust-attenuated flux in a given band, it is useful to immediately marginalize out the intrinsic magnitude,

$$p(m_v^{\text{obs}} | M_*, \psi_{\text{SFR}}) = \int p(m_v^{\text{obs}} | m_v^{\text{int}}) p(m_v^{\text{int}} | M_*, \psi_{\text{SFR}}) dm_v^{\text{int}}. \quad (18)$$

The integral can be evaluated analytically for our model pdfs;

$$p(m_v^{\text{obs}} | M_*, \psi_{\text{SFR}}) = \frac{1}{2} \frac{\text{erf}(y(m_v^{\text{obs}}, \pi/2)) - \text{erf}(y(m_v^{\text{obs}}, 0))}{\Delta m_v(\pi/2) - \Delta m_v(0)}, \quad (19)$$

where the denominator comes from the normalization of the uniform distribution, and the argument of the error functions is

$$y(m_v^{\text{obs}}, \theta) = \frac{\log(x_v(\theta)) - \mu^{(v)}}{\sqrt{2} \sigma_m^{(v)}}, \quad (20)$$

$$x_v(\theta) = \max\{\bar{m}_v^{\text{int}} + \Delta m_v(\theta) + b_v - m_v^{\text{obs}}, 0\}. \quad (21)$$

The dependence on M_* and ψ_{SFR} enters through \bar{m}_v^{int} and Δm_v .

It should be noted that this is a rather basic statistical model for intrinsic dust extinction. We have assumed that the optical depth in all galaxies follows the same mean relation, and (effectively) differs only in inclination angle. The other parameters of the model will surely also vary from galaxy to galaxy though, especially depending on its morphology – passive elliptical galaxies do not have separate bulge and disc components, for example. A simple extension to the model would be to choose a_{disc} separately for star-forming and passive galaxies, but we do not pursue this here. As such, this aspect of the model should be considered preliminary – it is able to fit the observations, as we shall see in Section 4, but any physical interpretation of the fits should proceed with caution.

4 MODEL CALIBRATION AND PARAMETER DEPENDENCE

A set of parameter values is needed as an input to the model. While best-fitting parameters for most of the conditional distributions in Fig. 1 are available separately in the literature, suitable parameter values can also be obtained by fitting the entire model to a set of diverse observational (or simulated) data. This has the advantage of ensuring consistency between the various components of the model. We will pursue the latter approach here, using a Monte Carlo sampling method to derive the joint posterior distribution of the model parameters given several data sets from galaxy surveys at different wavelengths.

In the first part of this section, we describe the data used to perform the fits: recent constraints on the radio LF of star-forming galaxies, and the LF of optically detected galaxies, all at $z \approx 0$. We also describe the sampling method, and cuts that were used on the data. We then report the parameter constraints that were

obtained, and analyse them for consistency using simple goodness-of-fit statistics. We also compare with the best-fitting values that were found for several of the model components independently by previous studies.

4.1 Input data

We use the following data to constrain the model:

(i) The LF of star-forming radio galaxies cross-identified in NVSS and 6dFGS (Mauch & Sadler 2007). This mostly constrains the SFMS and halo mass – stellar mass pdfs, and does not require a dust attenuation correction.

(ii) The *ugriz* optical LFs from GAMA (Loveday et al. 2012). These are corrected for extinction in the Milky Way, but not for the internal extinction in the target galaxies. We fit both the *g*- and *z*-band data to the full model; the *z* band is least affected by extinction, while the *g* band is strongly affected, allowing some of the parameters of the dust attenuation model to be constrained. Data from the other bands are not used in the general fits, but they are used to constrain the dust extinction amplitude parameter (conditioned on the best-fitting parameters of the general fit; see below).

(iii) The joint distribution of stellar mass, SFR, and intrinsic (zero extinction) optical magnitudes from the Guo et al. (2011) semi-analytic simulation. This is used to calibrate the optical magnitude conditional distributions only, as described in Section 3.5. The resulting best-fitting parameters are then fixed throughout the rest of the analysis.

These data sets were chosen because they are sufficient to constrain all of the key parameters of the model at $z \approx 0$. Other data sets or simulations could also have been included, but this is beyond the scope of the present work. Note that most of these data sets assume different background cosmologies; we applied corrections to convert them to our fiducial cosmology as appropriate.

We construct independent, approximate likelihoods for each data set, based on the binned data reported in the literature. The binned GAMA LFs are reported with symmetric error bars. We assume that these data are Gaussian distributed, and that the errors are independent, yielding a likelihood of the form

$$\log \mathcal{L}(\theta) = -\frac{1}{2} \sum_i \left(\frac{\Phi_i - \Phi(\theta)}{\sigma_i} \right)^2, \quad (22)$$

where i labels the magnitude bins of the LF, $\Phi = dn/dm$, and θ denotes the set of parameters for which the model LF is evaluated. The optical LF is calculated from the model as

$$\begin{aligned} \Phi(m_v^{\text{obs}}) &\equiv \frac{dn}{dm_v^{\text{obs}}} = \int d\psi_{\text{SFR}} \int dM_* \int d \log M_h \\ &\quad \times n(M_h) p(M_* | M_h) p(\psi_{\text{SFR}} | M_*) \\ &\quad \times p(m_v^{\text{obs}} | \psi_{\text{SFR}}, M_h). \end{aligned} \quad (23)$$

The NVSS/6dFGS radio luminosities are reported with asymmetric error bars. We were unable to find published posterior distributions for the data, and so we use an approximate likelihood for data with asymmetric errors from an unknown distribution, taken from Barlow (2003):

$$\log \mathcal{L}(\theta) \approx -\frac{1}{2} \sum_i x_i^2(\theta) (1 - 2A_i x_i(\theta) + 5A_i^2 x_i^2(\theta)); \quad (24)$$

$$x_i(\theta) = \frac{y_i - \hat{y}_i(\theta)}{\sigma_i}; \quad \sigma = \frac{\sigma_+ + \sigma_-}{2}; \quad A = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}, \quad (25)$$

Table 1. Summary of the best-fitting parameter values for the main components of the model. The best-fitting parameters for the optical magnitude relations are given in Table 2.

Stellar mass–halo mass relation, $p(M_\star M_h)$			
A_\star	Overall normalization	Equation (5)	0.018
β_\star	Slope at low-mass end		1.321
γ_\star	Slope at high-mass end		0.596
$\log_{10} M_1^\star$	Mass scale of transition in mean relation (M_\odot)		12.498
σ_1^\star	Overall dispersion amplitude at low masses	Equation (7)	0.557
σ_∞^\star	Dispersion at high-mass end		0.031
ξ_\star	Width of transition in dispersion [†]		4.250
$\log_{10} M_2^\star$	Mass scale of transition in dispersion (M_\odot) [†]		11.800
Galaxy type (passive fraction), $f_{\text{pass}}(M_\star)$			
α_f	Mass scale of transition (exponent)	Equation (8)	10.804
β_f	Determines width of transition		−2.436
ζ_f	Determines passive fraction at low mass		−1.621
Stellar mass–SFR relation (SFMS galaxies), $p(\psi_{\text{SFR}} M_\star)$			
α_{SFMS}	Overall normalization of mean relation (exponent)	Equation (9)	−0.077
β_{SFMS}	Power-law index of scaling with stellar mass		1.037
σ_{SFMS}	Dispersion of SFMS	Equation (10)	0.391
Stellar mass–SFR relation (passive galaxies), $p(\psi_{\text{SFR}} M_\star)$			
a_{pass}	Determines normalization of mean relation	Equation (11)	0.0011
σ_{pass}	Dispersion of passive sequence		0.029

Note. Parameters marked with (†) were fixed during the MCMC sampling procedure.

where y_i is the observed value, σ_\pm are the upper and lower asymmetric error bars on y_i , and $\hat{y}_i(\theta)$ is the model value for a given set of parameters. This limits to a Gaussian likelihood for symmetric error bars. Since the studies in question report asymmetric errors on the logarithm of the radio LF, we will work directly with $y_i = \log \Phi_i$ as the random variate. Again, this is an approximate form for the likelihood *in the absence of more information* about the posterior distributions. If we had access to the actual posteriors, this treatment would be unnecessary. The radio LF is calculated in our model as

$$\Phi(L_{\text{rad}}) \equiv \frac{dn}{d \log L} = \int d\psi_{\text{SFR}} \int dM_\star \int d \log M_h \times n(M_h) p(M_\star|M_h) p_{\text{SFMS}}(\psi_{\text{SFR}}|M_\star) \times L_{\text{rad}} p(L_{\text{rad}}|\psi_{\text{SFR}}), \quad (26)$$

where only the SFMS is used; passive galaxies are assumed to have been selected out.

The final log-likelihood is simply the sum of the log-likelihoods for each data set, i.e. two optical bands and the radio.

4.2 Sampling method

We use the `EMCEE` affine-invariant Markov Chain Monte Carlo (MCMC) sampler (Foreman-Mackey et al. 2013) to sample from the posterior distribution. We run this for 2000 samples for each of 128 workers, leaving 5500 samples after discarding burn-in and thinning the chains.

Seventeen (17) parameters were sampled by the MCMC (see Table 1 for a summary). We did not sample the parameters of the optical luminosity relation defined in Section 3.5, as this would allow too much freedom in the model. The posterior distributions of other parameters could shift if these were allowed to vary, although this would likely also lead to degeneracies. We did sample three of the dust attenuation parameters however (see Table 2). The starting position of the chains was chosen based on a rough visual fit to the input data, which aimed to find parameter values broadly similar

Table 2. Best-fitting values for the parameters of the optical magnitude model, including dust attenuation. The non-attenuation parameters were fitted to the $z = 0$ simulated galaxy catalogue of Guo et al. (2011). The attenuation parameters a_{disc} and β_τ were fitted to the g - and z -band GAMA LF data simultaneously with the non-optical model parameters (see below). The attenuation parameters τ_0 , λ_0 , and κ were fitted using a maximum likelihood method on the full five-band GAMA LF data (conditioned on all other model parameters).

Band	u	g	r	i	z
A_\star			3876.6		
β_\star			−0.000243		
c_\star			−1.011		
$A_\times^{(v)}$	−3.612	−2.700	−2.016	−1.726	−1.564
β_\times			−0.263		
γ_\times			0.290		
$c_0^{(v)}$	−27.318	−25.945	−25.284	−24.982	−24.810
$\mu^{(v)}$	−0.066	−0.054	−0.019	−0.009	−0.009
$\sigma_m^{(v)}$	0.281	0.252	0.247	0.253	0.272
τ_0			1.041		
λ_0			4977.2		
κ			0.004		
a_{disc}			1.429		
β_τ			5.478		

to the best-fitting values from the literature. This procedure was necessary to help avoid local maxima of the likelihood.

Simple prior bounds were chosen to keep the walkers in a physically reasonable region of parameter space. Certain features can be reproduced by allowing the SFMS to be too broad or too narrow for example, and the passive and star-forming sequences can switch position in the $\text{SFR} - M_\star$ plane if the amplitude parameters are not kept in check. Negative dispersion parameters should also be disallowed, as otherwise the pdfs become ill-defined. To avoid problems like these, we chose the following priors:

$$0.05 \leq \sigma_{\text{SFMS}} \leq 1 \quad (27)$$

$$0.01 \leq \sigma_{\text{pass}} \leq 2 \quad (28)$$

$$0.02 \leq \sigma_1^*, \sigma_\infty^* \quad (29)$$

$$0.001 \leq a_{\text{pass}} \leq 0.9 \quad (30)$$

$$11.6 \leq \log_{10} M_1^* \leq 12.5. \quad (31)$$

The last prior in this set was chosen to restrict the transition scale of the stellar mass–halo mass relation to be the same order of magnitude as the one found by Moster et al. (2010). Allowing this to be completely free often resulted in the walkers finding local minima with substantially different best-fitting parameters from others in the literature.

We identified the burn-in period by plotting the mean of the log-likelihood over all 128 workers as a function of sample number. The mean log-likelihood stops improving as the chains enter the maximum likelihood region, which was seen as a flattening of the plot after a certain number of steps. To thin the chains, we calculated the integrated autocorrelation time (Sokal 1997) for the chain from each worker as a function of thinning factor,

$$\tau_{\text{int}}(f) = \frac{1}{2} \sum_{\tau} \rho(\tau, f), \quad (32)$$

where $\rho(\tau, f)$ is the normalized autocorrelation function at lag τ and f is the thinning factor. We chose the smallest thinning factor that reduced the autocorrelation time below unity, and then combined the thinned chains from all workers into a single chain. As jumps between workers are allowed by ensemble samplers, the chains from each worker are correlated with one another, so this procedure does not fully decorrelate the samples. We verified that the sample means and standard deviations for a few parameters did not shift significantly when different thinning factors or burn-in periods were used though, or when different numbers of workers were kept in the final chains. The posterior distribution for all 17 parameters is shown in Fig. 4, and the processed chain is available to download from <http://www.philbull.com/ghost>.

Once a best-fitting model has been obtained from the MCMC, we then fit the frequency-dependent dust attenuation parameters τ_0 , κ , and λ_0 (see equation 16) to all five optical bands simultaneously, using a maximum likelihood method. The likelihood for the optical LFs is evaluated by varying these parameters while keeping all other parameters fixed to their best-fitting values. The results of this procedure are shown in Table 2 and Fig. 3.

4.3 Analysis of the model fits

The MCMC procedure described above results in a set of samples from the posterior distribution of the model and chosen data. The parameter values of the best fitting (i.e. maximum likelihood) model from the chains are summarized in Table 1. In this section, we provide a brief analysis of how well the global best-fitting model fits the input data: the optical and radio LFs from GAMA and NVSS, respectively. (The fit to the simulated optical magnitude data was discussed in Section 3.6.)

Fig. 5 shows the LF of star-forming radio galaxies at $z \simeq 0$ from the global best-fitting model, along with the NVSS data. The residuals are relatively small, with no large deviations across the full luminosity range. The model has a χ^2 of 30.1 for 10 data points however, which is formally a poor fit (although recall that we used an approximate, non-Gaussian likelihood function, and have

assumed that the errors are independent). This must be compared to the performance of other methods to model LFs though, which also tend to produce formally poor fits. For example, if we fit the phenomenological Schechter-like LF suggested by Mauch & Sadler (2007) to the radio LF only, using the same likelihood function, we obtain a best-fitting model with $\chi^2 = 18.6$ (for four free parameters), which gives a probability to exceed of 0.005 – a better, but still poor fit. We conclude that our model fit is acceptable for simple modelling purposes, due to the lack of significant deviations, but that it does not provide a complete, statistically acceptable description of the data.

The best-fitting model for the optical LFs is shown in Fig. 6. The residuals are again reasonably small, except for noticeable deviations at the extreme bright and faint ends in most bands, and a bump feature at $m_{\text{obs}} \approx -18$ in the u band. The goodness of fit for the bands that were used in the MCMC were again relatively acceptable but formally poor, with $\chi^2 = 39.5$ (49.2) and 34.1 (34.1) for the g and z bands, over 17 and 16 data points, respectively (where the numbers in parentheses denote the χ^2 after the frequency-dependent dust attenuation parameters have been fitted to all five bands). The u , r , and i bands, which were not included in the MCMC, have $\chi^2 = 195.2, 156.7, 87.8$ (161.0, 156.0, 85.0) respectively, with 17 data points each.

The deviations at the faint end of the LFs in Fig. 6 are likely due to completeness effects, where galaxies have gone undetected near the flux limit of the survey. While a completeness correction was applied to the data by Loveday et al. (2012), a systematic drop remains in the last few data points in all bands, which indicates a residual completeness systematic. We discarded the last three points in each band in the MCMC to avoid this skewing our results.

The deviations at the bright end are less severe, due to the larger error bars there. These could be caused by a model error; we do not explicitly include a bright starburst galaxy population for example, but this would contribute most significantly at the bright end. The deviation could also be explained by selection effects, e.g. due to bright galaxies being intrinsically rarer and thus more likely to be under sampled in surveys that cover limited volumes (as is always the case at $z \simeq 0$). Intrinsic dust attenuation also affects bright galaxies most significantly, so a model error or a selection effect related to this could also cause a discrepancy.

The bump feature in the u band is significant, even if the fit is otherwise (qualitatively) a good one. There is a slight hint of such a feature at around the same position (i.e. a little fainter than the faint-end transition in the LF) in all of the bands. The best-fitting model tries to explain this by allowing the LF of the sub-dominant passive sequence to form a relatively sharp peak at around this magnitude. This is achieved by driving the width of the passive sequence, σ_{pass} , to low values. Assuming that this bump is a real feature of the data, the failure of our u -band LF to reproduce it could be due to the mean optical magnitude model, equation (13). A systematic trend in the width and height of the bump in the passive LF can be seen in Fig. 6; both decrease towards bluer wavelengths. The data could likely be explained better if the bump was higher in the u band, which may be achievable with a slightly different set of optical magnitude relation parameters for this band, or by taking into account the structure of the residuals in the $\psi_{\text{SFR}} - M_*$ planes (see Fig. 2).

For the data set that was directly fitted for (i.e. the g - and z -band optical LFs and radio LF), we found an effective χ^2 of 103.7 for 43 data points and 17 free parameters, giving roughly 26 degrees of freedom when the data points are assumed independent. For the most optimistic hypothesis, that the data and error bars are correctly estimated and free of systematic errors, and that our model is the

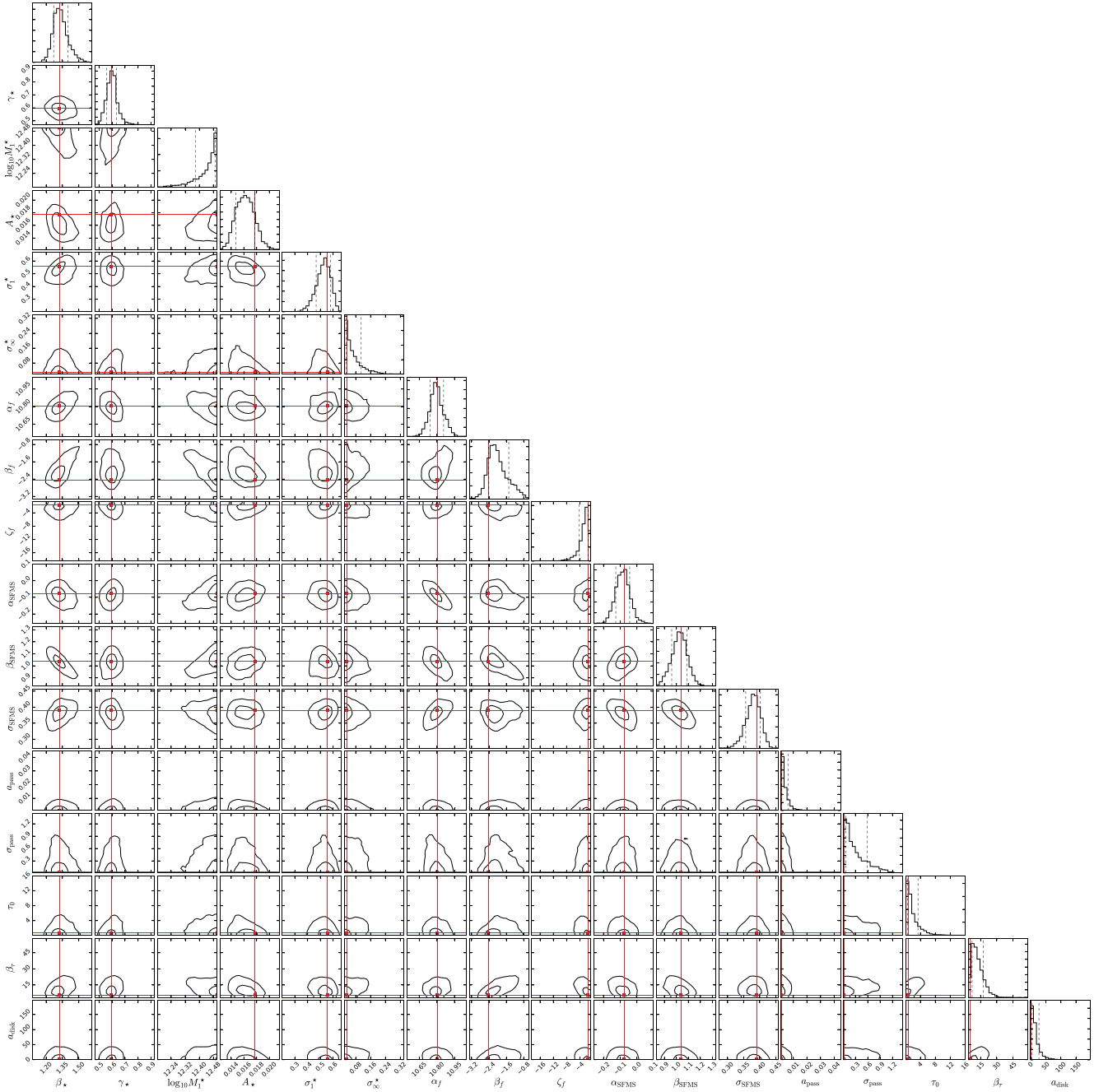


Figure 4. 2D posterior distributions of the 17 parameters that were varied in the MCMC. The contours are 1σ and 2σ bounds, and the marker shows the best-fitting model. The 1D marginal distribution for each parameter is shown on the diagonal. Some of the parameters with strongly non-Gaussian 1D marginal distributions are restricted by their prior bounds (e.g. σ_∞^*), while others simply prefer a logarithmic distribution (e.g. a_{disc}). We used the CORNER software (Foreman-Mackey 2016) to make this plot.

correct underlying description of the data, the probability to exceed this χ^2 is negligible ($p \lesssim 3 \times 10^{-11}$), implying that this hypothesis is incorrect. For the full data set, including the other three optical bands and the maximum likelihood-fitted frequency-dependent dust attenuation parameters, the total χ^2 is 515.5 over (effectively) 19 free parameters and 94 data points.

Our overall conclusion is that the data are qualitatively well fit by the model, which produces LF that closely follow the shape and amplitude of the LF data. If one assumes that the errors on the data are independent, the goodness of fit is technically poor however.

Numerous effects could contribute to cause this, one of which is likely to be the fact that the model is rather simple. Nevertheless, the results we obtained will be sufficient for many modelling purposes.

4.4 Consistency with previous results

In this section, we compare our global best-fitting model with a small selection of constraints on individual components of the model from the literature. Since we use only the radio and optical LFs, and SAM fits to the optical magnitude relation, to constrain the

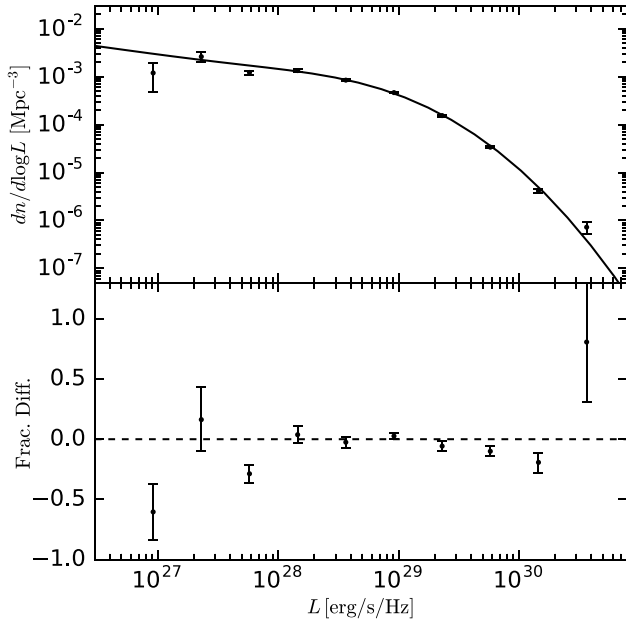


Figure 5. *Upper panel:* luminosity function of star-forming radio galaxies from the best-fitting model (solid black line) to the GAMA and NVSS data, with the latter data shown as black points. *Lower panel:* the fractional difference between the best-fitting model and the NVSS data points.

model, the constraints we place on other components are indirect. Comparing with direct constraints should then give us a good idea of the model’s consistency.

Fig. 7 shows the stellar mass function (SMF) for the SFMS and passive populations for the global best-fitting model, compared with data from SDSS/GALEX at $z \approx 0.1$ (Moustakas et al. 2013). Either the SFMS or passive SMF model predictions are a good fit to the data. The SFMS SMF has a similar shape to the data, and fits well at $M_* \sim 10^{10} M_\odot$, which is encouraging given that this is a prediction based on a fit to very different data. The amplitude of the high-mass end is systematically low however, by 2–3 standard deviations per data point, which accumulates to yield a poor overall goodness of

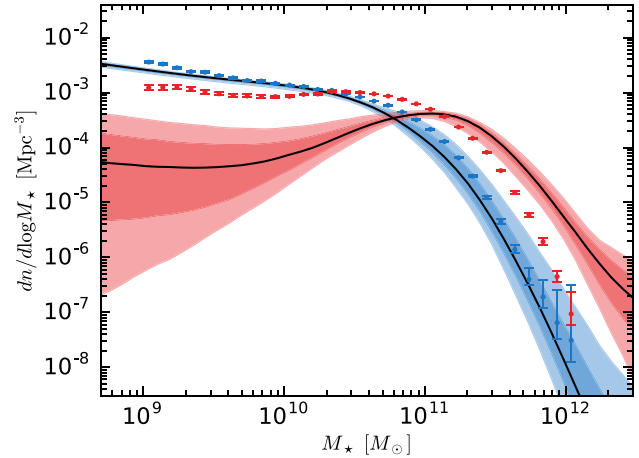


Figure 7. SMF for SFMS (blue) and passive (red) galaxies. Data points are from SDSS/GALEX at $z \approx 0.1$ (Moustakas et al. 2013). Solid black lines with coloured bands show the predictions of our global best-fitting model, including 68 per cent and 95 per cent credible regions from the MCMC. Our model fits did not use the SDSS/GALEX data, or any other SMF data; these curves are predictions.

fit. The passive SMF is a much worse fit, despite the relatively large uncertainty at high and low stellar mass from the MCMC.

This discrepancy could be caused by a number of different effects. For example, the categorization into star-forming and passive galaxies assumed by our model could differ markedly from the selection used by Moustakas et al. (2013), as the distinction is not particularly sharp. Fig. 8 shows the passive fraction, along with the SDSS/GALEX data and another model from the literature. Neither of the models nor the data agree with one another. An increase in the passive fraction at low mass would ease the discrepancy with the passive SMF, but make the discrepancy with the SFMS SMF worse. It is not clear that a globally acceptable fit could be found for both the SFMS and passive SMFs when taking the data at face value.

Residual systematic effects could also be present in the data. Fig. 4 of Moustakas et al. (2013) shows the total SMF from

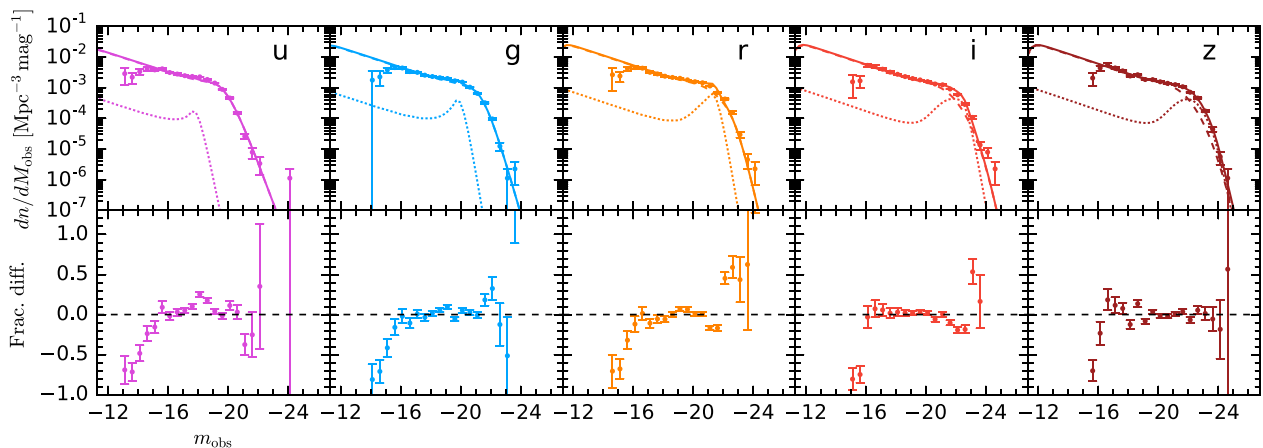


Figure 6. *Upper panels:* optical LFs in the *ugriz* bands at $z \approx 0$, fitted to data from GAMA (points with error bars) and the NVSS radio LF. Only the *g* and *z* bands were used in the fitting procedure. The solid coloured lines show the total LFs, including the dust attenuation correction; dashed lines show the LF from the SFMS only; and dotted lines show the passive galaxy LF. *Lower panels:* fractional difference between the best-fitting optical LF from our model and the GAMA data points.

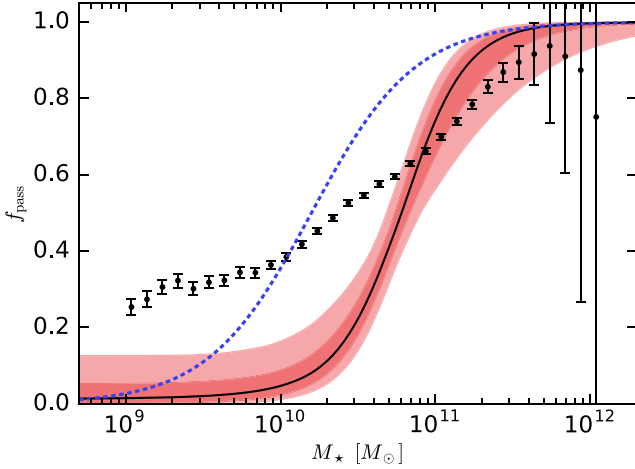


Figure 8. The predicted fraction of passive galaxies as a function of stellar mass. Red bands show the 68 per cent and 95 per cent credible intervals from our model, when fitted to the GAMA and NVSS optical/radio LFs. The purple dashed line shows the simpler relation assumed in Behroozi et al. (2013). The black data points show the passive fraction from SDSS/GALEX, derived by dividing the reported quiescent SMF by the sum of the quiescent and star-forming SMFs, and then estimating the errors using simple Gaussian error propagation.

SDSS/GALEX plotted alongside four previous determinations of the SMF from the literature. The difference between the curves is several times the error bars in most cases, suggesting that at least some of the five data sets are inconsistent with one another.

Finally, while the SDSS/GALEX error bars shown in Fig. 7 are small, there still appears to be relatively little scatter in the positions of neighbouring data points, indicating that the errors are correlated. We were unable to find information in Moustakas et al. (2013) to quantify this (e.g. a correlation matrix), but if the correlation is strong, this will substantially reduce the amount of independent information in the data, and thus the significance of the discrepancy with our model prediction.

Fig. 9 shows the best-fitting model for the mean SFR relation of the SFMS (equation 9), along with 68 per cent and 95 per cent credible intervals from our MCMC chains, and for a previous fit of a power-law relation to other data by Wang et al. (2013) (in the redshift bin $z = [0.2, 0.5]$). Roughly following the trend with redshift found in Wang et al. (2013), one would expect an extrapolation of their model to $z = 0$ to reduce its amplitude, but leave the slope relatively unchanged. The slope of our model is clearly much steeper. The comparison should only be performed above the completeness limit of the data that they used however, as denoted by the grey bands in Fig. 9. The errors on their fit are relatively large, and essentially cover the posterior of our relation above the completeness limit. Wang et al. (2013) do report that their best-fitting model has a reduced χ^2 of 0.2 though, which likely indicates that the error bars are overestimated.

The number density distribution in the $\psi_{\text{SFR}}-M_*$ plane is plotted in Fig. 10 for both the SFMS and passive population. These are compared with the distribution at $z = 0$ from the Guo et al. (2011) simulation. The SFMS aligns very well with the one found in the simulation, with essentially the same slope and a similar width. Our model appears to overestimate the density of galaxies at high ψ_{SFR} and M_* , but this could also be due to under sampling in the simulation (which has a relatively small box size of $62.5 h^{-1} \text{Mpc}$). The passive distribution is entirely different from the one in the

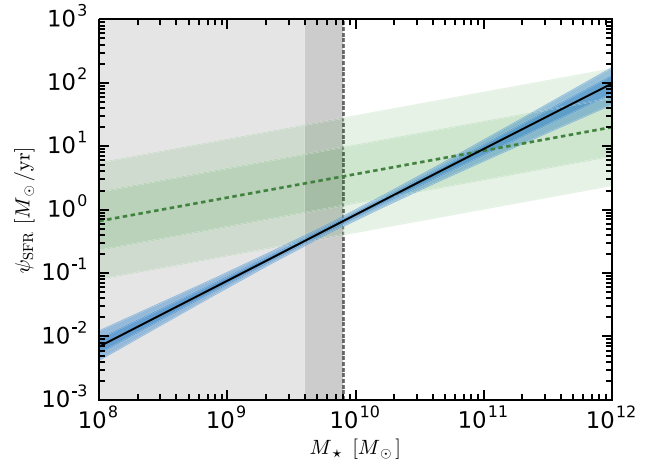


Figure 9. The mean SFR as a function of stellar mass. The blue bands show 68 per cent and 95 per cent credible intervals for our model from the MCMC. The green dashed line shows the best-fitting Wang et al. (2013) relation in the redshift range $z = [0.2, 0.5]$ (corresponding to $\alpha = -3.14$ and $\beta = 0.37$) along with the reported errors on the fit (green bands). The grey shaded regions show stellar masses that lie below the completeness limit of some (or all) of the data used by Wang et al. (2013) to fit their model.

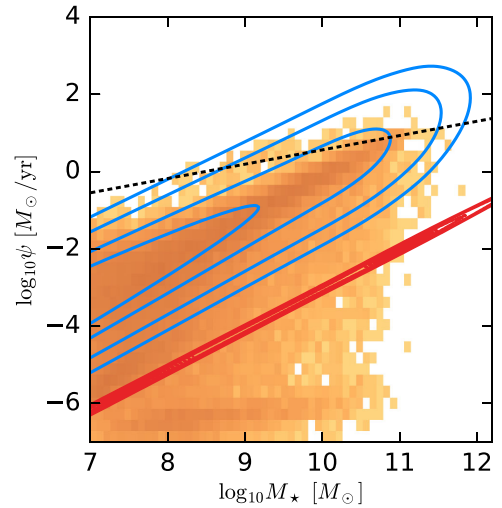


Figure 10. Predicted number density distribution in the M_* – ψ_{SFR} plane, $dn/d\log M_* d\log \psi_{\text{SFR}}$, for SFMS (blue) and passive (red) galaxies. The model shown is the best-fitting model to the GAMA and NVSS optical/radio LF data. Contours are shown at values of $\{10^{-3}, 10^{-4}, 10^{-6}, 10^{-8}\} \text{Mpc}^{-3}$ (SFMS) and $\{10^{-4}, 10^{-6}, 10^{-8}\} \text{Mpc}^{-3}$ (passive), respectively. The density plot (orange) shows the same quantity measured from the Guo et al. (2011) semi-analytic mock catalogue at $z = 0$. This simulation was not used in the fits, except to define the optical magnitude parameters in equation (13). The dashed line is the same as in Fig. 9.

simulation however; the simulated one has no sequence-like structure, while our best-fitting model prefers a very narrow sequence. One can imagine that a much larger value of σ_{pass} would have been able to approximately reproduce the simulated result if this was preferred by the data, but it is not. As discussed in Section 4.3, part of the reason for this preference might be the need to reproduce a small, narrow feature in the optical LFs. The various SAMs for the passive population also greatly differ from one another (see Section 3.3), and so the one that we chose (and/or the one in the

simulation) may simply be inconsistent with the data. This could also cause the large discrepancy with the passive SMF that was seen in Fig. 7.

In summary, our model appears to reproduce previous results reasonably well, yielding qualitatively good fits to the optical and radio LFs, reasonable overlap with a previous determination of the SFMS, and only a mild underestimate of the SMF of star-forming galaxies. The passive sequence of the best-fitting model appears to be strongly discrepant with simulations and some previous results however (although we have noted that different simulations give a broad spread of predictions for the shape of the passive population in the SFR– M_* plane). A detailed investigation to construct a more accurate model for the passive population is left for future work.

4.5 Joint luminosity functions

We now give an example of the kind of multiple-tracer prediction that the model is designed to calculate. Given two samples of galaxies in different wavebands, it is useful to quantify what fraction of the galaxies can be detected in both bands simultaneously. This can be used to predict how efficiently sources can be cross-matched between surveys, for example, or for calculating correlated shot noise in the cross-correlation of the surveys. The basic quantity required for these calculations is the joint LF, $dn(L_1, L_2)/d \log L_1 d \log L_2$. As an example, we will calculate the joint LF for one optical band and the 1.4 GHz radio continuum emission from star-forming galaxies. This can be written as

$$\begin{aligned} \frac{dn(L_{\text{rad}}, m_v^{\text{obs}})}{dL_{\text{rad}} dm_v^{\text{obs}}} &= \int d\psi_{\text{SFR}} \int dM_* \int dM_h \frac{dn(M_h)}{dM_h} \\ &\times p(L_{\text{rad}}, m_v^{\text{obs}} | M_*, \psi_{\text{SFR}}) \\ &\times p(\psi_{\text{SFR}} | M_*) \cdot p(M_* | M_h). \end{aligned} \quad (33)$$

All of these distributions were defined above, with the exception of the joint luminosity/magnitude pdf, which we model as

$$p(L_{\text{rad}}, m_v^{\text{obs}} | M_*, \psi_{\text{SFR}}) = p(L_{\text{rad}} | \psi_{\text{SFR}}) \cdot p(m_v^{\text{obs}} | M_*, \psi_{\text{SFR}}). \quad (34)$$

This assumes that the two types of emission are linked only by their common dependence on ψ_{SFR} and M_* , i.e. that they are independent when we condition on SFR and stellar mass. We have also used the fact that the radio luminosity depends only on SFR. In fact, our model assumes a deterministic relationship between SFR and L_{rad} (equation 12), so that

$$p(L_{\text{rad}} | \psi_{\text{SFR}}) = \delta^D(\psi_{\text{SFR}} - \psi_{\text{SFR}}^{\text{rad}}(L_{\text{rad}})), \quad (35)$$

where δ^D is the Dirac delta function. equation (33) then simplifies to

$$\begin{aligned} \frac{dn(L_{\text{rad}}, m_v^{\text{obs}})}{d \log L_{\text{rad}} dm_v^{\text{obs}}} &= L_{\text{rad}} \int dM_* \frac{dn(M_*)}{dM_*} p(\psi_{\text{SFR}}^{\text{rad}}(L_{\text{rad}}) | M_*) \\ &\times p(m_v^{\text{obs}} | M_*, \psi_{\text{SFR}}^{\text{rad}}(L_{\text{rad}})), \end{aligned} \quad (36)$$

where we have also changed variables to $\log L_{\text{rad}}$.

With the joint LF in hand, it is straightforward to evaluate the number density of galaxies that are detected by both optical and radio surveys. Assuming luminosity and magnitude limits L_* and m_* respectively, we have

$$\begin{aligned} n(L_{\text{rad}} \geq L_*, m_v^{\text{obs}} \leq m_*) \\ = \int_{\log L_*}^{\infty} \int_{-\infty}^{m_*} \frac{dn(L, m_v)}{d \log L dm_v} d \log L dm_v. \end{aligned} \quad (37)$$

The fraction of optically detected galaxies that also have radio counterparts (and vice versa) is then

$$f(\text{rad} \in \text{opt}) = \frac{n(L_{\text{rad}} \geq L_*, m_v^{\text{obs}} \leq m_*)}{n(L_{\text{rad}} \geq 0, m_v^{\text{obs}} \leq m_*)} \quad (38)$$

$$f(\text{opt} \in \text{rad}) = \frac{n(L_{\text{rad}} \geq L_*, m_v^{\text{obs}} \leq m_*)}{n(L_{\text{rad}} \geq L_*, m_v^{\text{obs}} \leq \infty)}, \quad (39)$$

where the denominators are simply the total number density of optical/radio galaxies in each survey respectively, regardless of whether they have radio/optical counterparts or not.

Fig. 11 shows the joint LF for optical bands and a general direct SFR tracer (like radio continuum) as predicted by our model, for the best-fitting parameter values described above. As expected, galaxies that are most actively star forming are brightest in the optical, although there is not a 1:1 correspondence even in the case where dust attenuation is neglected. This is simply a consequence of the optical magnitudes also tracing the stellar mass of the galaxies; the spread in M_* for fixed ψ_{SFR} and the scatter in the optical magnitude pdf broaden the joint LF. The correlation is tighter in the bluer bands, where the emission is dominated by younger stars that trace recent star formation.

With attenuation included, the picture becomes more complicated. For the bluest bands, in which attenuation is the strongest, a branch of high-SFR galaxies appears that spreads broadly down to fainter optical magnitudes. The spreading effect is reduced at lower SFR however, mostly because our model of attenuation scales with stellar mass; since ψ_{SFR} also scales with M_* , lower SFR corresponds, on average, to lower stellar mass and thus weaker attenuation. Note that the number density of galaxies in this branch is relatively low, and that the main effect of attenuation is the shifting of the bright end of the joint LF to fainter optical magnitudes.

5 DISCUSSION

We have presented a modular, analytic statistical model that connects host DM halo properties to bulk properties of the galaxies that reside within them, which are then linked to observables such as galaxy luminosity across a range of bands. This model was motivated by the existence of various scaling relations between these quantities, and the need to model multiple tracer populations across wavelength regimes when analysing forthcoming multi survey large-scale structure data sets.

We showed that conditional distributions representing various scaling relations – many of which were already known in the literature – can be combined in a consistent way to simultaneously model the LFs of low-redshift galaxies in the optical and radio, with good precision. Consistency is achieved by jointly fitting the model parameters to multi wavelength data, instead of combining relations that were separately fitted to various data sets. We also reported on the uncertainties and correlations between model parameters that resulted from this fitting procedure, using the results of an MCMC sampling procedure.

As presented here, our model predicts only a limited set of galaxy properties that are important for future surveys. We did not attempt to model (e.g.) the neutral gas content or emission from nuclear activity, even though these will be important for radio surveys, for example. We posit that many such properties can be included as straightforward extensions to the model, by adding suitable conditional distributions to the probabilistic graph shown in Fig. 1.

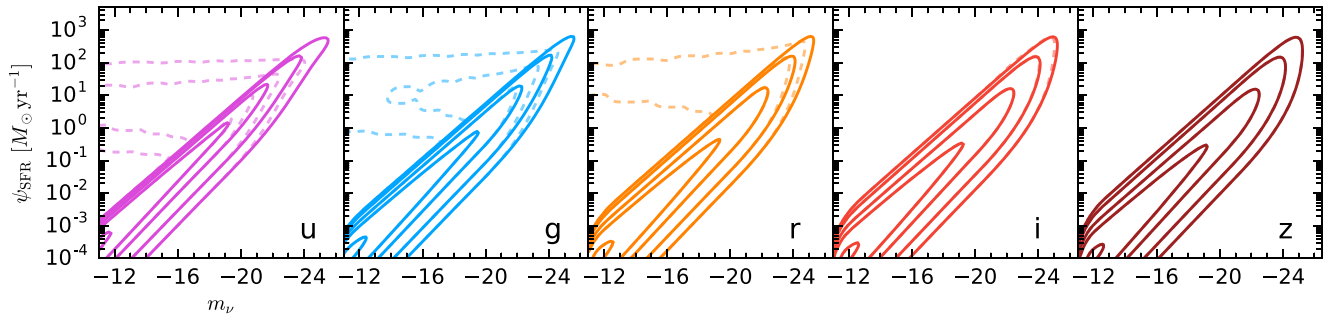


Figure 11. Joint LF of galaxies on the SFMS at $z=0$, for a tracer of the SFR (e.g. radio continuum at 1.4 GHz) and five optical bands. The solid contours show the LF without dust attenuation in the optical, while the dashed lines show the result with attenuation. The values of the contours are $dn/d\log \psi_{\text{SFR}} dm_{\nu} = \{10^{-8}, 10^{-6}, 10^{-4}, 10^{-3}, 10^{-2}\} \text{ Mpc}^{-3}$ (from outermost to innermost).

This will generally involve finding appropriate scaling relations between galaxy properties and then promoting them to (parametrized) statistical distributions.

Many such scaling relations have been seen observationally or in simulations, or result from simple physical arguments. Properties of AGN such as their radio and X-ray luminosity may be modelled through scaling relations involving the mass of their central black hole, stellar mass, or halo mass, for example (Ferrarese 2002; Fontanot et al. 2011; Aird et al. 2012). Relatively simple models of the neutral hydrogen (HI) content of galaxies also exist (e.g. Obreschkow et al. 2009; Bagla, Khandai & Datta 2010; Villaescusa-Navarro et al. 2014; Padmanabhan & Refregier 2016), often involving a scaling between HI mass and halo mass only. Aside from using the simple star-forming/passive categorization to assign disc/elliptical labels to the galaxies, more sophisticated morphological properties can also be modelled using scaling relations (e.g. see the treatment in Schreiber et al. 2016). Adding these properties into the model, plus luminosity relations for other bands, is left for future work, but it should be clear that extensions like this are supported in a natural way.

While our model does explicitly include per-band optical magnitudes, including dust attenuation, there is some further structure in the relationships between different bands that is not modelled – namely, colours. Galaxy colours are commonly used as selection criteria in surveys, as structures exist in colour–magnitude diagrams that can reliably distinguish between different types of galaxy. The existence of a red sequence in the colour–magnitude diagram (Bell et al. 2004) can be used to identify the large red galaxies that typically lie at the centre of galaxy clusters, for example (e.g. Rykoff et al. 2014). Our model fails to reproduce the red sequence however. This is because the optical pdfs [equation (14)] are assumed independent; the magnitudes in different bands are linked only through the mutual dependence of the mean relations [equation (13)] on stellar mass and SFR. Sequences in the colour–magnitude diagram also require correlations in the *scatter* between two bands, however. This could potentially be implemented by replacing equation (14) with a multivariate lognormal pdf for all bands, with $\sigma_m^{(v)}$ replaced by a full covariance matrix, thus at least allowing (log-)Gaussian correlations in the scatter.

While sufficient for some applications, this would also limit the model. Real galaxies have complex optical SEDs that depend on a host of other factors, in addition to bulk properties like stellar mass and SFR. The star formation history of the galaxy and its IMF are two such ingredients, both of which were used in the semi-analytic simulations from which our optical magnitude relations were calibrated. In future work, it would be desirable to remove

the dependence of this part of the model on SAMs, and all of the assumptions that underlie them. One way of achieving this would be to build the optical magnitudes from bandpass integrals over distributions of template galaxy spectra, such as those derived from the SPS models of Bruzual & Charlot (2003). This method has been used in a number of galaxy mock catalogue generation algorithms, as well as in MCMC analyses of galaxy spectra (e.g. Leja et al. 2017). The downside is that it introduces significant extra complexity, still requires a number of assumptions (e.g. about SFR history), and destroys the analytic nature of our model. A middle ground might be to use sets of empirical galaxy spectrum templates, such as those generated by some photometric redshift estimation methods (e.g. Masters et al. 2015), although this also adds significant complexity.

The model is incomplete in another important respect – we have calibrated it using only $z \simeq 0$ observations. The redshift dependence of several of the constituent conditional distributions has been studied previously (e.g. see Behroozi et al. 2010 for the redshift dependence of the stellar mass–halo mass relation), and so redshift-dependent parametrizations are readily available. These simply add more parameters to the model that can be constrained with additional data at other redshifts. Other components are new to this work, however. We have tentatively confirmed that some of these components have redshift dependences that are amenable to simple parametrizations, by performing fits on semi-analytic simulations (e.g. for equation 13). A full treatment, including constraining the model from higher redshift data, is left to future work.¹

In future, the model will also need to account for clustering observables. This will likely require several modifications to the early parts of the chain of conditional distributions that link halo mass to galaxy type, stellar mass, and SFR. The current setup would not be able to explain the observed SFR-dependence of clustering, for instance (cf. Tinker et al. 2013; Zu & Mandelbaum 2016; Coil et al. 2017). Models that are able to successfully reproduce observed clustering signals generally divide galaxies into ‘centrals’ and ‘satellites’, and permit more than one galaxy per host halo. These features could be implemented by adding a conditional distribution for the satellite mass function after the halo mass function (see Fig. 1), and allowing later steps of the chain to also be

¹ Ideally, this would result in every component having a redshift dependence that is well described by simple, few-parameter extensions to the model. In the worst case is should still be possible to find a new set of best-fitting parameters for every redshift slice, however.

conditioned on the central versus satellite classification. Scaling relations that differentiate between centrals and satellites in (e.g.) the stellar mass–halo mass relation already exist in the literature (e.g. Moster et al. 2010), and would be straightforward to adopt, at the expense of adding several additional parameters. Later parts of the chain that depend only on stellar mass and SFR would likely remain unchanged. A proof of concept is left to future work, however.

We have also neglected several observational effects that are important in modelling the observed distribution of galaxy luminosities. A small fraction of galaxies are strongly lensed, for example, which enhances their apparent luminosity. This causes a magnification bias (Kochanek 2006), which can be especially important at the bright end of the LF, where intrinsically bright objects may be rarer than the lensed population of fainter objects. As such, our LFs should be convolved with a lensing magnification pdf (e.g. Marra, Quartin & Amendola 2013) to account for this effect.

Finally, we note that a shortcoming of this kind of modelling is the inevitable proliferation of parameters. As the model is extended to account for increasingly complex observables, new model components (and therefore new parameters) will need to be added. The question is whether the growing parameter space will eventually become unwieldy, at least compared with just using SAMs or other more physics-based modelling approaches. This very much depends on what the model will be used for, and how the parameters will be constrained. In this paper, we fixed a number of the optical magnitude parameters, based on the outputs of an SAM. This ‘calibration’ approach helps to keep the number of degrees of freedom in check while allowing the model complexity to be increased significantly, and is legitimate if one has confidence that the input parameter values (whether taken from a theoretical model or external data) are sufficiently realistic and well constrained to be fixed in this way. This is effectively done by SAMs and hydrodynamical simulations too, which often have hidden ‘fixed’ degrees of freedom, such as libraries of stellar templates, fixed stellar population models, and sub-grid physics.

In general, we expect that current galaxy surveys and simulations are sufficiently information-rich to allow significantly more model parameters to be constrained, or sensibly fixed a priori, than were considered here. Some parameters will likely remain ill-constrained, or degenerate with one another, but this is not necessarily problematic if the target observables can still be calculated sensibly when these are marginalized over. The question is then whether the user of the model finds the added complexity acceptable and/or useful, or whether other approaches are more appropriate for their intended application. It is hard to say exactly how much more complexity would be required to achieve our goal of accurately modelling the one- and two-point functions of several different tracers, but we are hopeful that it can be managed by adding only a few additional components, as discussed above.

This process will be aided by the modular nature of the model, which allows us to easily compare several alternatives for each of the constituent conditional distributions. Through statistical tests, such as Bayesian model selection procedures, one can then choose components that most faithfully describe various observations, while penalizing overly flexible models that lack predictive power. These can then be swapped into the default configuration of the model, without requiring it to be completely rebuilt.

The type of model we have presented here fills a gap between detailed, computationally expensive galaxy modelling methods like semi-analytics and hydrodynamical simulations on the one hand, and single-population halo models (e.g. HOD models) on the other. It has sufficient structure to allow consistent modelling of multi-

ple galaxy populations across multiple wavelengths, but is simple enough that it can be explored analytically, allowing MCMC analyses to be performed without extensive computational resources. An open-source reference implementation, written in PYTHON, and making use of the NUMPY and SCIPY (Jones et al. 2001) packages, is available from <https://github.com/philbull/ghost>.

ACKNOWLEDGEMENTS

I am grateful to O. Doré, B. Hensley, E. Huff, A. Merson, P. Serra, M. Viero, H.-Y. Wu and an anonymous referee for valuable discussions, comments and encouragement, and to A. Brown for valuable work on a summer project involving this model. I gratefully acknowledge use of computing resources at the University of Oslo. PB’s research was supported by an appointment to the NASA Postdoctoral Program at the Jet Propulsion Laboratory, California Institute of Technology, administered by Universities Space Research Association under contract with NASA.

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